Detecting Capital Market Convergence Clubs*

Thanasis Stengos† M. Ege Yazgan‡ Fuat C. Beylunioglu§

February, 2017

Abstract

In this study, we propose a new method to find convergence clubs that combine pairwise method of testing convergence with maximal clique algorithm. Unlike many of those already developed in the literature, this new method aims to find convergence clubs endogenously without depending on priori classifications. We use our method to study convergence among different capital markets as captured by their respective indices. Stock market convergence would indicate the absence of arbitrage opportunities in moving between the different markets as they would all present investors with similar risks. Furthermore, stock market convergence would be a precursor to GDP convergence as these economies would be bound by similar (possibly unobservable) common factors that affect long run macroeconomic performance.

Keywords: Stock Market Convergence, Convergence Clubs, Maximal Clique Algorithm.

JEL Classification: C32, O47.

*This work has been produced as a part of the research project (project no: 113K757) supported by The Scientific and Technological Research Council of Turkey (TUBİTAK). Ege Yazgan and Thanasis Stengos would like to acknowledge financial support from TUBİTAK and Thanasis Stengos also from SSHRC of Canada.

†University of Guelph, email: stengos@uoguelph.ca

‡Istanbul Bilgi University, email: ege.yazgan@bilgi.edu.tr

§Istanbul Bilgi University, email: can.beylunioglu@bilgiedu.net

1 Introduction

Even though the issue of convergence is an important area of active research in much of the literature of empirical economic growth, it has only been recently that it has been given some importance and attention in other areas of empirical economics, see Abbott and De Vita (2013), Fritsche and Kuzin (2011), Abbott et al. (2012), Kim and Rous (2012), Apergis and Padhi (2013), Yilmazkuday (2013) and Ikeno (2014) to name a few. We will try to show that the methodology that is central to the empirical investigation of economic growth convergence can be also quite useful in analyzing convergent behavior in finance and in particular in capital markets. The implications of such behavior would be important for portfolio selection, spillover effects and diversification strategies for global investors. The prevalent approach to studying convergence in the macro/growth context is to analyze the convergent behavior of two economies (or the lack of it) by examining whether their output gap is stationary with a constant mean, irrespective of whether the individual country's output is trend stationary and/or contains a unit root, see Pesaran (2007).[1] It is worth noting that this analysis only considers the binary process of convergence (or lack of it) for pairs of countries and it has nothing to say how if al all, there is convergence to a common cluster. In other words the analysis so far, has mainly analyzed the issue of convergence between "country-pairs", but is mainly silent on how to proceed to classify countries as belonging to a common "country club". In a recent paper Stengos et al. (2016) have tackled this issue using a particular clustering methodology, the maximum clique algorithm widely used in graph theory from the computer science literature, see Bron and Kerbosch (1973) and Konc and Janezic (2007). The properties of the above clustering algorithm have been recently examined by Beylunioğlu et al. (2016) using Monte Carlo simulations and it is shown to perform fairly well in classifying units correctly into their respective clusters, when compared with other algorithms in the literature, see for example Hobijn and Franses (2000).

In this paper we will examine the issue of convergence in the context of a different economic environment, that of capital markets, relying on a similar empirical strategy outlined above. We

[1] An extension of this $I(1)/I(0)$ analytical set up to allow for a more general long memory framework has been used in the literature by Dufrénot et al. (2012) and Stengos and Yazgan (2014b). Using all available pairs to construct the pair wise differences without a benchmark country, Stengos and Yazgan (2014b,a) produce classifications based on the estimation of d, the long memory parameter. They find that even within a richer fractional integration classification analysis, the simpler $I(1)/I(0)$ framework can adequately capture the convergence patterns of all the considered countries confirming the findings of Pesaran (2007).

will apply the methodology discussed above to different indices of capital markets and analyze whether they converge to multiple steady states and whether we have the emergence of "convergent capital market clubs". We will be applying unit root tests to the pairwise differences of stock market indices of different capital markets, with convergence reached when the proportion of rejections obtained from the pairwise unit root tests is greater than a certain threshold. A capital market index represents a weighted average of all traded stock in the capital market in question and as such it offers a summary of the overall return/risk exposure of the capital market in question. An implication consistent with the efficient market hypothesis would suggest that the index cannot be beaten in terms of performance by market participants and convergence to a common cluster of a group of different indices would imply that these capital markets are similar in terms of risk exposure. As such there could be no hedging of risks between these markets and no diversification possible. On the other hand, lack of clustering and convergence would imply that these capital markets offer different levels of risk and as such they could offer potential diversification opportunities for investors. Rather than testing a-priory grouped country clusters, such as capital markets in emerging markets or OECD countries the method explores all convergent groups in a list of N indices that is subjected to pairwise convergence tests within a $I(0)/I(1)$ framework. We will use this approach as an endogenous cluster formation method for the all available stock market indices and not simply of small predefined groups.

The paper is organized as follows. We first proceed by presenting a literature review that examines systems of capital market indices. Then we will proceed to introduce our method that combines unit root testing within a $I(1)/I(0)$ framework with an the maximum clique approach from the computer science graph theory to establish a set of statistical criteria for cluster formation. We will then apply the proposed methodology to a set of capital market indices and present the results and finally we will conclude.

2 Literature Review

Even though the empirical work on convergence within the framework of capital markets has been quite recent, there have been some early theoretical works about integration or limitations of international capital markets.[2] The literature has mainly looked at the issue of integration between capital markets by employing vector autoregressive (VAR) methods and/or methods that look for the presence of cointegrating relationship. In the first category we have papers such as Morana (2008) who investigated co-movements among the G7 countries using stock market returns and a number of macroeconomic variables such as real output, inflation, oil prices, nominal short- and long-term rates, nominal money balances, and the real effective exchange rate employed to proxy unobserved economic fundamentals and to control for within country and across countries financial markets interactions. Demian (2011) studied Central and Eastern European countries integration also by using a VAR approach, whereas Caporale and Spagnolo (2011) employ a trivariate VAR-GARCH(1,1) model to check the interdependence of the three Central and Eastern European countries, Chech republic, Hungary, Poland with both Russia and the UK markets. The empirical findings suggest that there is significant co-movement (interdependence) of these markets with both the Russian and the UK ones. Furthermore, whilst the introduction of the euro has had mixed effects, EU accession has resulted in an increase in volatility spillovers between the three countries considered and the UK (contagion). Finally, Rockinger and Urga (2000) utilized an AR(1) model with time-varying parameters and possibly heteroskedastic residuals to test whether stock market predictability has decreased over time finding that the Hungarian market always satisfies weak efficiency, while for the Czech and Polish markets, there is convergence toward efficiency. On the other hand, a constantly significant level of predictability characterizes the Russian market. Scheicher (2001) uses a VAR-GARCH model on returns and found that both regional and global shocks affect returns but only regional innovations affect volatility. Maneschiöld (2006) applies cointegration tests to analyze the existence of long-run relationships among Baltic stock markets and other major international stock markets, including the United States, Japan, Germany, the United Kingdom, and France. Bivariate and multivariate cointegration tests indicate a common trend linking Latvia to European markets, whereas the German market dominates this long-run relationship. The results suggest that international investors can obtain diversification benefits given a long-term investment horizon because of the low degree of integration between the Baltic and international capital markets. Égert and Kočenda (2007) looks at Central and Eastern Eu-

[2]See Grauer et al. (1976), Harvey (1989), Ferson and Harvey (1993, 1994), Dumas and Solnik (1993) for fully integration; Black (1974), Stulz (1981), Errunza and Losq (1985), Eun and Janakiramanan (1986) for different models.

ropean data to examine return and volatility co-movements and finds short-term spillover effects both in terms of stock returns and stock price volatility. Onay (2006) and Chelley-Steeley (2005) also use Eastern European data to examine the integration of these capital markets with the more advanced European and US markets, whereas papers that look mainly at volatility linkages are Kim et al. (2005), Bekaert and Harvey (1997))and Fratzscher (2002).

Papers that have looked more directly to the issue of cointegration and convergence of capital market series are Bruno et al. (2012) who adopt β and σ convergence approaches developed in the economic growth literature by Baumol (1986), Sala-i Martin (1996) and Mankiw et al. (1990) to analyze OECD and G7 deposits, debt securities, shares and insurance products. They find strong evidence supporting the existence of β-convergence of shares and insurance products, hence confirming the notion of integration of capital markets in the developed world. In contrast, mixed results are obtained for debt securities and deposits due to differences across countries in the weight that national public debt plays and in the role of banks. Another paper in this context is Narayan et al. (2011) who uses similar convergence analysis to examine convergence of stock markets in different environments. Eun and Lee (2010) studied risk return characteristics of 17 developed stock markets between 1974 and 2007 and also examined convergence of the risk-return characteristics of a sample of 14 emerging markets, finding that they have been converging rapidly toward those of developed markets in recent years. Apergis et al. (2011) employs σ-convergence based Phillips and Sul (2007) method to investigate the dynamics of international equity markets for OECD markets and find that international equity markets do not form a homogeneous convergence club. Gilmore et al. (2008) analyzes short-term and long-term co-movements between developed European Union (EU) stock markets and those of the three Central European (CE) countries which recently joined the EU and examine the extent to which the co-movements of the CE countries and the two developed EU markets can be related to common unobserved factors over time. Yunus (2013) also employs the recursive cointegration test developed by Hansen and Johansen (1999) and further refined by Brada et al. (2005), Rangvid (2001) and Mylonidis and Kollias (2010), to assess financial contagion, whereas Bley (2009) found results that revealed the time-varying nature of the financial market integration process finding that more integration between 1998 and 2006 and considerably less after that period as return behavior has been changing and stock markets within the Euro zone have been starting to drift apart. In a similar spirit, Awokuse et al. (2009) studies selected Asian emerging markets and three major stock markets (Japan, UK and US) by using rolling cointegration analysis of Johansen (1991), whereas Pascual (2003) does the same to examine long-run co-movements in the UK, French, and German stock markets and so does Mylonidis and Kollias (2010) who investigate European economic integration (Germany (DAX), France (CAC), Spain (IBEX) and Italy (FTSE MIB)).

3 Methodology

As mentioned above, our approach will combine pairwise testing with the maximal clique algorithm from computer science graph theory introduced by Özkan et al. (2016). As it will be shown below, our approach based on the maximal clique method is a top down method that finds all the set of countries satisfying the definition of a club.

The logarithmic gap Z_t^{ij} between the stock market index series of country i and j at time t expressed in common currency can be expressed as follows:

$$Z_t^{ij} = \ln(S_t^i) - \ln(S_t^j/E_t^e) = \beta_t + \varepsilon_t \sim I(d), \quad i = 1, ..., N, \ i \neq j, \ t = 1, ..., T, \tag{1}$$

where T is the length of time interval, N refers the number of markets, S_t^i and S_t^j denote the stock market index series of i and j country. E_t^e denotes the currency of index j in terms of currency of index i. It is the expected exchange rate at time t formed on the information available at the previous period. As usual, ε_t stands for the disturbance term, which is assumed to follow an $I(0)$ process and β_t represents, possibly, time varying cross country risk. Finally $d \in \{0, 1\}$ represents the degree of integration of the common currency index gap series, Z_t^{ij} and it can take the values 1 or 0. Since the ratio of the series are either stationary or non-stationary, that will determine if the pair is convergent or not. Under constant arbitrage across integrated capital markets and a stationary $I(0)$ risk profile across country pairs, Z_t^{ij} would follow a $I(0)$ stationary process. Under these circumstances the log-difference of the common currency indices i and j series will be drifting together and in that case it would be appropriate to assert that they are convergent. On the other hand, if β_t were to be a nonstationary process, this would indicate that log-difference of these common currency market index series is nonstationary as they would be drifting apart over time, indicating that they are not converging.

Rearranging Z_t^{ij} in equation (1) yields,

$$U_t^{ij} = \ln(S_t^i) - \ln(S_t^j) = \beta_t + \varepsilon_t - \ln(E_t^e) \sim I(d), \tag{2}$$

where U_t^{ij}, stands for the log-difference between the stock market index series of i and j. Differently from Z_t^{ij}, the possible nonstationarity of U_t^{ij} may originate either from β_t being $I(1)$and/or $\ln(E_t^e)$ being $I(1.)$. The latter can be thought to depend on the evolution of the cross country gaps of some relevant macro indicators ($\mathbf{F}_t$) such as interest rates, aggregate incomes etc. In that case we analyze conditional convergence, a concept that is common in the empirical growth literature, so that equation (2) takes now the form

$$U_t^{ij} = \mu_{ij} + \tau_{ij}(\mathbf{F}_t^i - \mathbf{F}_t^j) + e_t^{ij}, \tag{3}$$

In that case we can use the residuals, $\hat{e}_t^{ij}$, instead of the stock market indices gaps, U_t^{ij} themselves in the analysis. In our applications we make use all of the quantities in equations (1), (2) and (3), i.e. Z_t^{ij}, U_t^{ij} and $\hat{e}_t^{ij}$ for examining the convergence properties.

According to our approach, if one tests for convergence of a group of N units (countries, stock market indices etc.) all $N(N-1)/2$ pairs are subjected to unit root testing, using Augmented Dickey Fuller (ADF) tests.[3] Pesaran (2007) showed that, if a group of N units are non-convergent, the rejection rate of the null hypothesis of non-stationarity ($H_0 : Z_t \sim I(1)$) calculated by $N(N-1)/2$ tests is equal to the nominal size of the individual tests, i.e. the probability of Type 1 error. More specifically, it is shown that under the null hypothesis of N units being non-convergent, the rejection rate of individual tests converges to the nominal size, α, as N and $T \to \infty$, even though individual tests are not independent cross- sectionally. Thus, in order to reject convergence of N stock market indices, it is enough to show that the proportion of rejections over $N(N-1)/2$ tests is smaller than the significance level of individual tests. In that case for example, if the significance level is 5%, the proportion of rejections must not exceed 0.05.[4] To summarize, rejection rates higher than a given significance level in a given application would imply evidence in favour of the

[3]The ADF testing regression has a drift and rejection of the unit root is accompanied by an insignificant trend to establish convergence.

[4]No doubt, nominal size of the tests may differ from the significance level. In applications, the power of the tests used relative to size distortions should be given attention. Another matter to be attentive to is the fact that the rejection rate would converge to α in the limit and that of course would not be the case if N and T are relatively small in a given application.

convergence hypothesis (rejection of the null of non-convergence). On the other hand, rejection rates lower or close to the employed significance level will provide evidence for the non-rejection (validity) of the non-convergence null hypothesis.

3.1 Maximal Clique Method for Finding Convergence Clubs

The maximal clique method that we present in this subsection, combines the maximal clique algorithm of graph theory with the previously described pairwise convergence tests of $H_0 : Z_t \sim I(1)$. Rather than testing a priory grouped country clusters, the method explores all convergent groups in a list of N markets (countries) that was previously subjected to pairwise convergence tests.

The method consists of two steps. First, all possible pairwise differences of N markets (countries) are subjected to stationarity tests where the null denotes a unit root process. If the rejection rate obtained from $N(N-1)/2$ tests falls above the significance level, that would be evidence against the non-convergence null hypothesis and the list of N countries will be taken to be form a convergent group. If this club involves all examined countries, then all countries are said to be convergent and we do not go any further in seeking out the presence of convergence clubs. However, it is very unlikely that, by examining all markets as a single group one will find evidence of convergence for all with pairwise testing. If convergence of a subgroup of countries is found via pairwise method, it can be said that this group constitutes a convergence club. Nevertheless, if a subgroup of all countries is found convergent via pairwise method, then it can be said that this subgroup constitutes a convergence club. The main challenge is to find a method to determine this subgroup rather than relying on a-priori classifications. In the second step we undertake this task.

Suppose $\mathbb{A}$ denotes the set of all countries. Hence by definition, the cardinality of $\mathbb{A}$ is equal to N; mathematically if $\#()$ denotes the cardinality, we have $\#(\mathbb{A}) = N$. Moreover, suppose that $\mathbb{E}$ is a subset of U. In this case, in order $\mathbb{E}$ to be a convergence club, all binary combinations obtained with elements of $\mathbb{E}$ should satisfy the pairwise stationarity tests. Hence, since $\#(\mathbb{E}) = M < N$, the rejection rate obtained via all $M(M-1)/2$ pairs should fall above the significance level.

In the second step, from the $N(N-1)/2$ test results, the objective is to find a class of subsets $\mathbb{G}$ for which all subsets, e.g. $\mathbb{E}$, satisfy the pairwise convergence property. Mathematically speaking, let $\mathbb{G}$ denotes the class of all subsets satisfying the desired pairwise (stationarity) property. Then the problem is

$$\mathbb{G} := \{\mathbb{E} : \vee i, j, \quad i \neq j, \ \in \mathbb{E}, \ t\left(Z^{ij}\right) = 1\} \tag{4}$$

where Z^{ij} represents the differenced quantity pairs under consideration, $t(\cdot)$ is the test result of the series in the bracelet and takes the value of 1 for a convergent pair, i, j and 0 otherwise.[5] Hence, the problem can be expressed as

$$\arg\max_{\mathbb{G}} \{ \#(\mathbb{E}) : \ \ \mathbb{E} \in \mathbb{G}\}. \tag{5}$$

In graph theory terms, countries become vertices and convergent country pairs defines edges. The set of all vertices and edges constitutes an undirected graph. An example of such a graph is depicted in Figure 1 where the $N = 11$ countries are represented by numbers in each vertices. If an undirected graph has edges between all vertices then the graph is said to be complete. If there is a subset of an undirected graph having all properties of a complete graph, the subset is so called a *clique*. Therefore, in our case, all convergence clubs of a country list can be expressed as cliques. In Figure 1, countries 1-6, 6-9, 10-11, and their all subsets form the set of all convergence clubs (all *clique*s), referred as $\mathbb{G}$ above.

On the other hand, solving the problem defined in (5) above is known as finding *maximum cliques*, i.e. finding the convergent club(s) with maximum number of elements. Figure 2 shows the *maximum clique* of undirected graph in Figure 1. However, in our case, focusing only on maximum cliques may lead us to disregard smaller clubs that are obviously meaningful entities for the purpose of our analysis. The notion of *maximal clique*, offered by graph theory, helps to overcome this difficulty. A *maximal clique* can be defined as a clique that is not a subset of any other clique. Thus, detecting all *maximal cliques* in a group of N countries provides us the list all convergence clubs excluding their subsets. In other words, the set of all convergence clubs,

[5]Notice that in order to satisfy the property explained above, for $\mathbb{E}$ to be a convergence group, all pairs $i, j \in \mathbb{E}$, $i \neq j$ should satisfy the convergence property. This requires the rejection rate to be one which is a much more stringent condition, since it does not allow for Type 2 error. One can relax this condition by allowing rejection rate up to a given level. It is worth noting that allowing the rejection rate up to a given level will have an impact on how much Type 2 error allowed by the unit root test would carry over in club formation. It could be argued that since in general unit root tests suffer from low power using a rejection rate close to one can be interpreted as a precautionary step to confront this issue.

$\mathbb{C}$ is a subset of $\mathbb{G}$ which is not a subset of any other $\mathbb{E} \in \mathbb{G}$. Figure 3 illustrates (the set of) all convergence clubs ($\mathbb{C}$) for the group of countries and pairwise test results in Figure 1. As is illustrated in Figure 3, in this example, the country represented by number "6" belongs to two different convergence clubs. These countries (vertices) counted in different clubs more than once may contain important information.

Listing all maximal clique can be too hard from a computational point of view. The computational complexity of solution to maximal clique problem is known NP-Complete whose brute-force solution requires $2^N - \binom{N}{2} - N - 1$ trials per clique. First, Bron and Kerbosch (1973) developed an algorithm to solve the problem in exponential time. In this study we list all maximal cliques by employing a variant of Bron and Kerbosch algorithm proposed by Eppstein et al. (2010), an algorithm benchmarks the problem to sparseness of the graph that ends in linear time.

3.2 Other club formation methods

An alternative club formation method proposed in the literature is due to Hobijn and Franses (2000) (henceforth HF) who developed a panel data based approach for testing convergence. Contrary to the early attempts that relied on a two stage method that first assigns membership to a group and then considers whether this assignment is satisfied by the data, HF classifies countries into clusters if they satisfy some criterion (desired convergence property). They clustered countries into subgroups by applying multivariate stationarity tests to panels consisting of pairwise differences of income per capita series. A different approach was proposed by Kapetanios (2003); Chortareas and Kapetanios (2009) (2003, 2008) who developed a method that is designed to endogenously classify stationary and nonstationary series by sequentially reducing the size of the null. That occurred by removing series with the most evidence against the unit root null, classifying these series as stationary. The stopping point is when the unit root null does not reject, such that all the remaining regions are declared nonstationary.[6]

[6] Using the HF methodology, Corrado et al. (2005) extended this method by allowing subgroups to vary over time and applied it to European regional sectoral data of agriculture, manufacturing and services. Corrado and Weeks (2011) extended the sequential HF approach to account for short time panels by using a bootstrapping modification and applied their method to study regional European convergence. A similar approach is advanced using the notion of $\sigma-$convergence by Phillips and Sul (2007) who developed an algorithm based on a log-t regression approach that clusters countries with a common unobserved factor in their variance. In the convergence literature, $\sigma-$ convergence as opposed to $\beta-$ convergence deals with the reduction in the variance of the cross country income distribution over time, see Quah (1996).

The current paper aims at developing a convergence analysis technique of cluster (club) formation that relies on pairwise testing both in the simpler $I(0)$ or $I(1)$ framework as in Pesaran (2007) combined with the maximal clique algorithm widely used in graph theory from the computer science literature, see Bron and Kerbosch (1973) and Konc and Janezic (2007). Rather than testing a-priory grouped country clusters, the method explores all convergent groups in a list of N countries that was previously subjected to pairwise convergence tests within a $I(0)/I(1)$ or a long memory framework.

The pairwise method combined with the maximal clique algorithm proposed here and HF are both seeking convergence by searching similarities in movements of outcomes in the process of time. To this end both methods expect all pairs in a club to move around zero or a constant, so that they are stationary. However there is a crucial difference in the approaches as well as the treatments of pairs. In the construction of a single club, HF is a bottom up method that forms the clubs by adding countries one by one while the maximal clique method, by employing the pairwise method as outlined above is a top down method that finds all the set of countries satisfying the definition of a club. Other than clustering, there is a substantial difference in testing convergence. To determine whether a set of countries is convergent, HF applies multivariate stationarity test to panels comprised of consecutive pairwise difference series set elements and confirms convergence if the null hypothesis of stationarity of the panel is not rejected. However, the panels do not include all possible pairwise differences but only differences of consecutive pairs. For example, if we want to test the convergence of countries 1,2,3 and 7, a panel consisting of Z^{12}, Z^{23} and Z^{37} is subjected to the test, and if stationarity cannot be rejected the panel is then augmented to include additional difference series. If then in this "augmented" panel the stationary null is rejected, then these four countries are said to be convergent. On the other hand, our proposed pairwise method depends on a different definition of clubs, so that for M countries to be convergent, we need to achieve rejection of the null of a unit root for all $M(M-1)/2$ pairs. Hence, in order for the list of countries in the previous example to form a convergence club, the rejection rate of $4(4-1)/2 = 6$ pairs from unit root tests should exceed some significance level.[7]

In a recent paper Beylunioğlu et al. (2016) have compared the above methods by an extensive Monte Carlo simulation study and they have found the pairwise method combined with the max-

[7]As emphasized in Footnote 5 above, in our applications the rejection rates are taken as unity as we do not allow any Type 2 error.

imal clique algorithm to produce more reliable results than HF is terms of correctly identifying country clubs (clusters). The data generating mechanism used in the simulations was a latent factor model that generated clubs with members with common factors. The results suggest that in terms of accuracy the ADF-maximum clique augmented pairwise method does quite well in detecting correctly the presence of clubs or clusters of countries. This gives us confidence that using the above method to real data would provide us with useful insights about club formations of different groups of similar characteristic as far as economic activity is concerned.

3.3 Stock Market Convergence

We applied the above analysis of club formation to stock market indices using data at both daily and monthly frequencies. The data are obtained from Bloomberg Terminal and International Financial Statistics. The number of countries included in the daily data analysis is equal to 19.[8] This number reduces to 18 in the analysis of monthly figures as a result of the exclusion of Ireland from the list to keep the monthly data at a reasonable length. The choice of countries is dictated by their data availability. While the daily observations include the period of November 1st, 2010 to February 29th, 2014, the monthly figures cover a longer period from January 1st, 2008 to October 31st, 2016.[9]

The analysis for Z_t^{ij}, the stock market index log-differences in common currency, is conducted only for daily figures, by replacing E_t^e by E_t, i.e. ex-post realized rates on daily basis. On the other hand the conditional convergence analysis with U_t^{ij} and $\hat{e}_t^{ij}$ is carried out by both daily and monthly data, where $\hat{e}_t^{ij}$ is taken to be the OLS residuals from equation (3), where $\mathbf{F}_t^i - \mathbf{F}_t^j$ represents the cross country interest rate differentials in the daily data, and income (industrial production index) and interest rate differentials in the monthly data.[10]

As mentioned above our approach is based on conducting unit root tests on a group of N stock market indices, where all $N(N-1)/2$ pairs are subjected to unit root testing. Following

[8]These countries are Austria, Belgium, Bulgaria, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Poland, Portugal, Spain, UK, USA, Sweden, and Turkey. Basically we included all the country whose data are available. The problem mainly lies in the non-availability of interest rate data at daily frequency.

[9]Due to several missing interest rate data, in some specific years, at daily frequency in countries like Austria (2009), Bulgaria (2008-10), Finland (2009-10), Poland (2009), Ireland (2008-9), Turkey (2014), the calender coverage of daily data is shortened relative to monthly data to maximize the number of countries included in the analysis.

[10]Nominal interest rates, 2-year government bonds yields, i_t are calculated as $i_t = \ln(1 + i_t)$, incomes are represented as the log levels of industrial production indices. The absence of relevant data does not permit us to use more macroeconomic indicators such as money stock levels in the monthly data.

Pesaran (2007), in order to reject the non-convergence null hypothesis (find evidence in favour of convergence) among N stock market indices, it is enough to show that the proportion of rejections over $N(N-1)/2$ tests is larger than the significance level of individual tests.[11] To summarize, rejection rates much higher than a given significance level in a given application would imply evidence in favour of the convergence hypothesis. On the other hand, rejection rates lower or close to the employed significance level will provide evidence for the non-rejection (validity) of the null hypothesis of non-convergence. Unit root testing has been done with Augmented Dickey Fuller (ADF) tests. Beylunioğlu et al. (2016) provide evidence that the maximal clique method based on ADF testing has good small sample properties in detecting single and multiple clubs when compared with other algorithms in the literature, see for example Hobjin and Franses (2000).

Table 1 summarizes the results of the analysis using Z_t^{ij}, U_t^{ij}, $\hat{e}_t^{ij}$ and shows the number of clubs found by our method with different number of members of clubs (M). For example, with daily data over the period of 2008-2015, the analysis for Z_t^{ij} indicates 5 clubs with 2 members, a single club with 3 members, and no club with 4 or more members. The results suggest that for U_t^{ij} the convergence patters are similar to those of Z_t^{ij} with 3 clubs of size 2, a single club of size 3, without an indication of any club with size 4 or larger. However, the results for $\hat{e}_t^{ij}$ are very different indicating a much larger degree of convergence with the varying number of clubs from 2-6 members. This is to be expected as these residuals capture the common elements among the pairs of indices after the factors that could have caused non-convergence such as interest rate differentials have been accounted for. To see the implication of smoother data we repeat the analysis with monthly series for the period of 2008-2016 by excluding Z_t^{ij}. We also use monthly observations of industrial production in addition to interest rates in the creation of $\hat{e}_t^{ij}$. As the results shows smoother data leads to larger degree of convergence with more clubs as in the case of $\hat{e}_t^{ij}$ above.

Table 2 displays the number clubs that each countries belong. For example, in the daily data analysis, the maximal clique method finds Austria as a member of 1, 0 and 4 distinct clubs for Z_t^{ij}, U_t^{ij}, and $\hat{e}_t^{ij}$ respectively. The results for Z_t^{ij}, indicate that while Netherlands, Portugal, Spain belong to 2 different clubs, Bulgaria, Canada, Denmark, Finland, France, Ireland, Italy,

[11] For example, if the significance level is 5%, the proportion of rejections must exceed 0.05. However, as we mentioned above (see footnote 5) the rejection rates that we use are unity in our applications to account for possible low power of the unit root tests.

Poland, Turkey stand as countries that do not belong to any club. On the other hand Austria, Belgium, Germany, Japan, Sweden, UK and USA are found as a member of a single club only. As above, while the results of Z_t^{ij} and U_t^{ij} are very similar, those of $\hat{e}_t^{ij}$ indicate completely different pattern. Looking at the results of $\hat{e}_t^{ij}$ Turkey is a member of 9 different clubs followed by Belgium, Germany and Portugal with 8 memberships each. A discernible difference between the results of monthly and daily data appears also clearly in Table 2, especially in the case of Bulgaria.

Figures 4 - 7 illustrate some examples of clubs for stock market convergence found by our method. Figure 4 shows France, Belgium and Portugal as a club of 3 found in the analysis of both U_t^{ij} and Z_t^{ij} in the daily data. Figure 5 and 6 exemplify Netherlands, France, Portugal, UK, Spain, Japan; and Portugal, Denmark, Austria, Germany as clubs with $M = 6$ and $M = 4$ from the analysis of daily data with $\hat{e}_t^{ij}$. Similarly in Figure 7 Netherlands, Belgium, UK, Canada make up a club of size 4 in the monthly data covering the whole period of 2008-2016 for $\hat{e}_t^{ij}$. These clubs suggest that there are certain unobservable common factors among its member in terms of having similar trade pattern and exposure to financial risk.

The main finding of our study is that we have identified convergent stock market clubs that are linked together by (possibly unobservable) common factors. A capital market index represents a weighted average of all traded stock in the capital market in question and as such it offers a summary of the overall return/risk exposure of the capital market in question. According to the efficient market hypothesis, an index cannot be beaten in terms of performance by market participants and convergence to a common cluster of a group of different indices would imply that these capital markets are similar in terms of risk exposure. Hence, within the group of stock markets that form the given club, the characteristics of risk and returns would be similar and as such arbitrage opportunities and risk diversification would not be very likely. Furthermore, as these common factors are the main determinants of stock market returns of firms producing output in the member countries that form the given club, that would reflect of course the overall macroeconomic performance of the economies in question, at least in the long-run. As a consequence, it is expected that stock market convergence would mimic the convergence in their respective GDP's per capita for these countries, something that is one of the main research areas of the empirical growth literature.

4 Conclusions

In this paper we examined convergence of capital markets by using a methodology that relies on unit root testing. Within a $I(1)/I(0)$ framework we examined different pairs of capital market indices and we analyzed whether they converged to multiple steady states, that is whether we encountered the emergence of "convergent capital market clubs". Applying unit root ADF tests to the pairwise differences of stock market indices we reach convergence when the proportion of rejections obtained from the pairwise unit root tests equals one. A capital market index represents a weighted average of all traded stock in the capital market in question and as such it offers a summary of the overall return/risk exposure of the capital market in question. Hence, convergence to a common cluster of a group of different indices would imply that these capital markets are similar in terms of risk exposure and as such there could be no hedging of risks between these markets and no diversification possible. On the other hand, lack of clustering and convergence would imply that these capital markets offer different levels of risk and they could offer potential diversification opportunities for investors. Rather than testing a-priory grouped country clusters, our methodology explores all convergent groups in a list of N indices that is subjected to pairwise convergence tests within a $I(0)/I(1)$ framework and produces an endogenous cluster formation mechanism for all available stock market indices. In this context cluster membership implies market efficiency as there would be no arbitrage opportunities to be hedged among member capital market indices.

Tables and Figures

Table 1: Stock Market Convergence: $\hat{e}_t, U_t, Z_t \sim I(0)$, 5% significance level, Overall

Data	Model	# 2	# 3	# 4	# 5	# 6
Daily	$Z_t \sim I(0)$	5	1			
	$U_t \sim I(0)$	3	1			
	$\hat{e}_t \sim I(0)$	3	3	5	3	5
Monthly	$U_t \sim I(0)$	4	3	2	0	2
	$\hat{e}_t \sim I(0)$	2	14	7		

Table 2: Stock Market Convergence: $\hat{e}_t, U_t, Z_t \sim I(0)$, 5% significance level, Membership Counts

	Daily			Monthly	
Countries	Z_t	U_t	$\hat{e}_t$	U_t	$\hat{e}_t$
Austria	1	0	4	1	4
Belgium	1	1	8	4	5
Bulgaria	0	0	2	10	15
Canada	0	0	0	1	3
Denmark	0	1	4	0	2
Finland	0	0	1	4	0
France	0	0	6	1	2
Germany	1	1	8	2	3
Ireland	0	0	2	NA	NA
Italy	0	0	3	1	2
Japan	1	1	4	1	2
Netherlands	2	0	5	2	2
Poland	0	1	3	1	5
Portugal	2	2	8	1	3
Spain	2	1	4	2	5
Sweden	1	0	1	1	5
Turkey	0	0	9	3	7
UK	1	1	4	1	5
USA	1	0	4	1	4

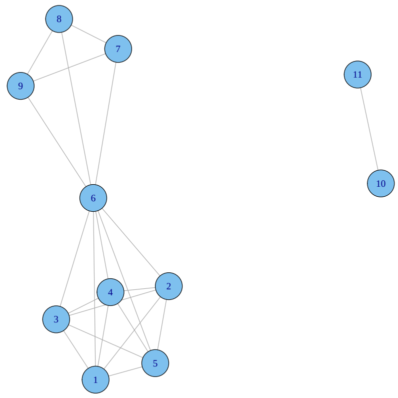

Figure 1: A sample undirected graph

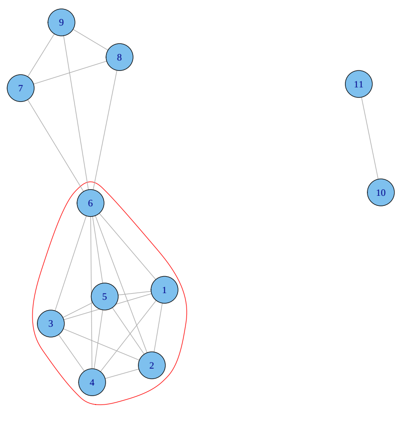

Figure 2: A sample maximum clique

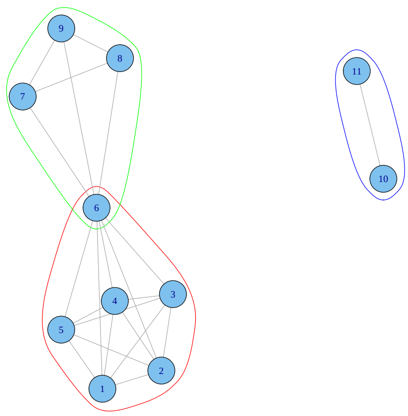

Figure 3: A sample of maximal cliques

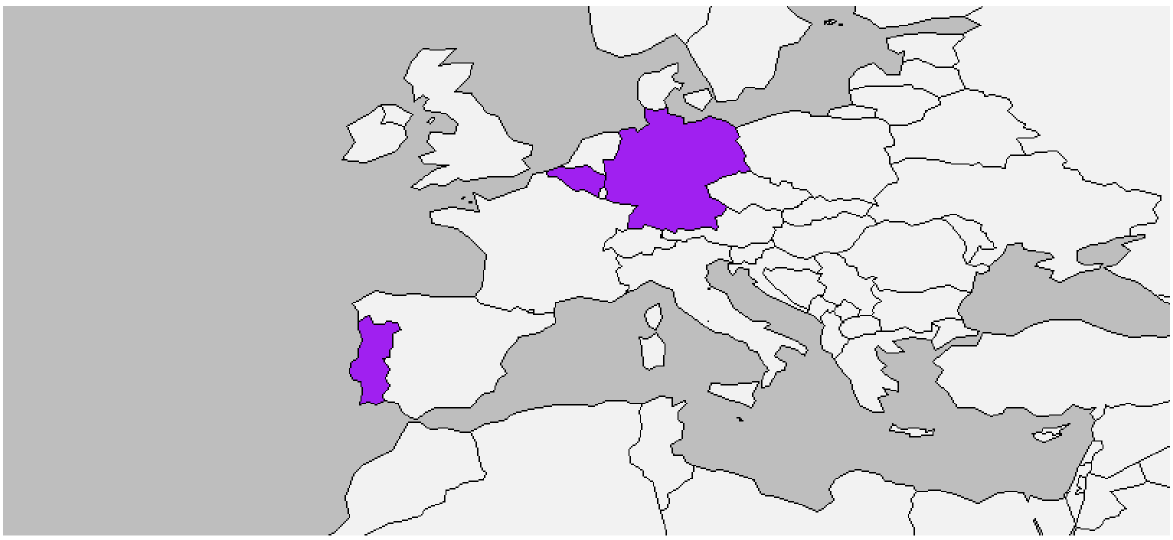

Figure 4: Illustration of a Club, Daily Data, $H_0 : Z_t \sim I(0); H_0 : U_t \sim I(0)$

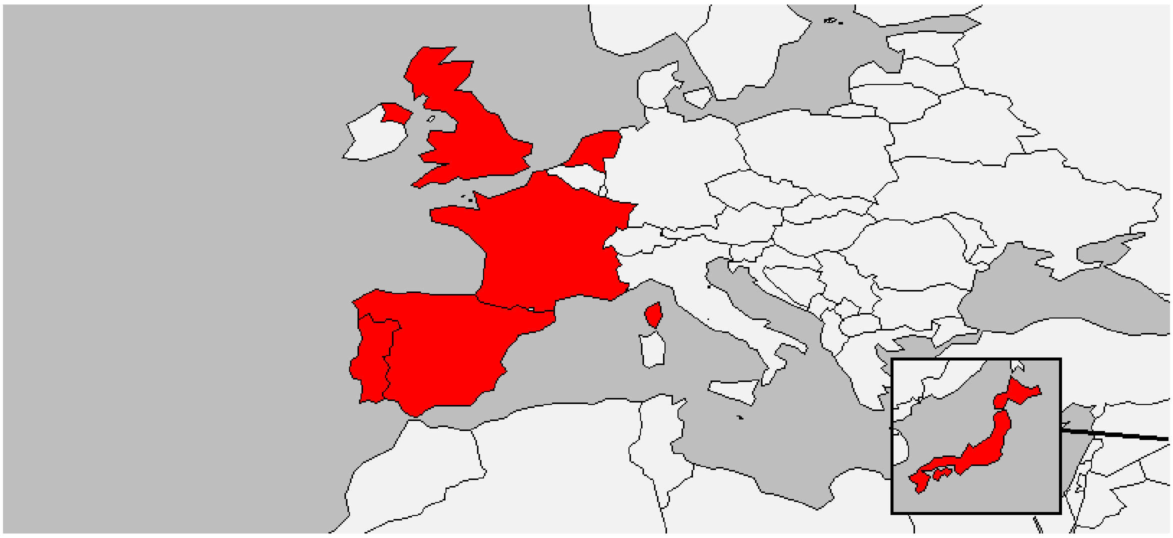

Figure 5: Illustration of a Club, Daily Data, $H_0 : \hat{e}_t \sim I(0)$

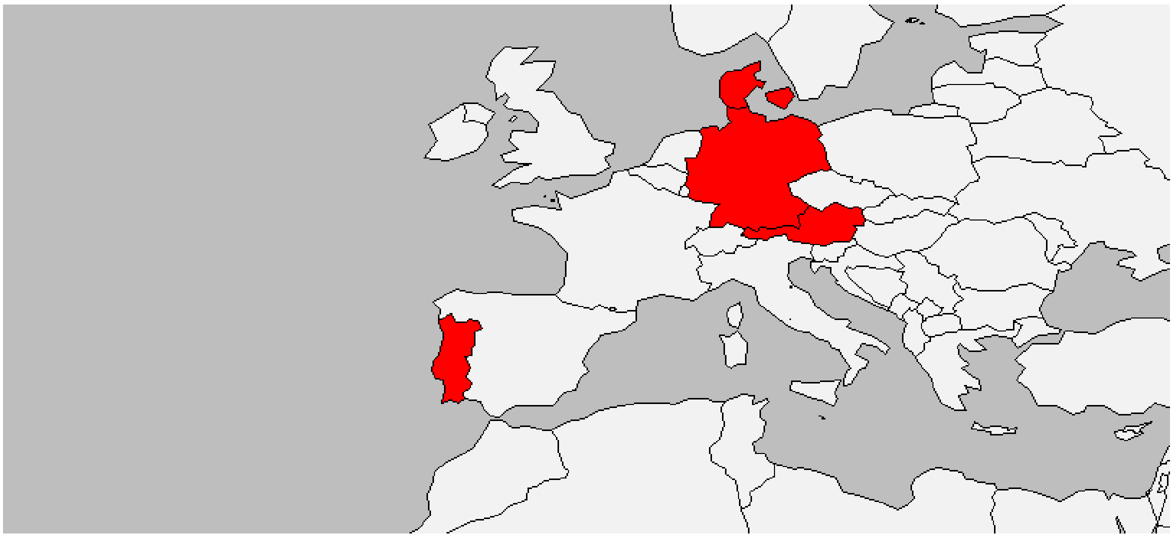

Figure 6: Illustration of a Club, Daily Data, $H_0 : \hat{e}_t \sim I(0)$

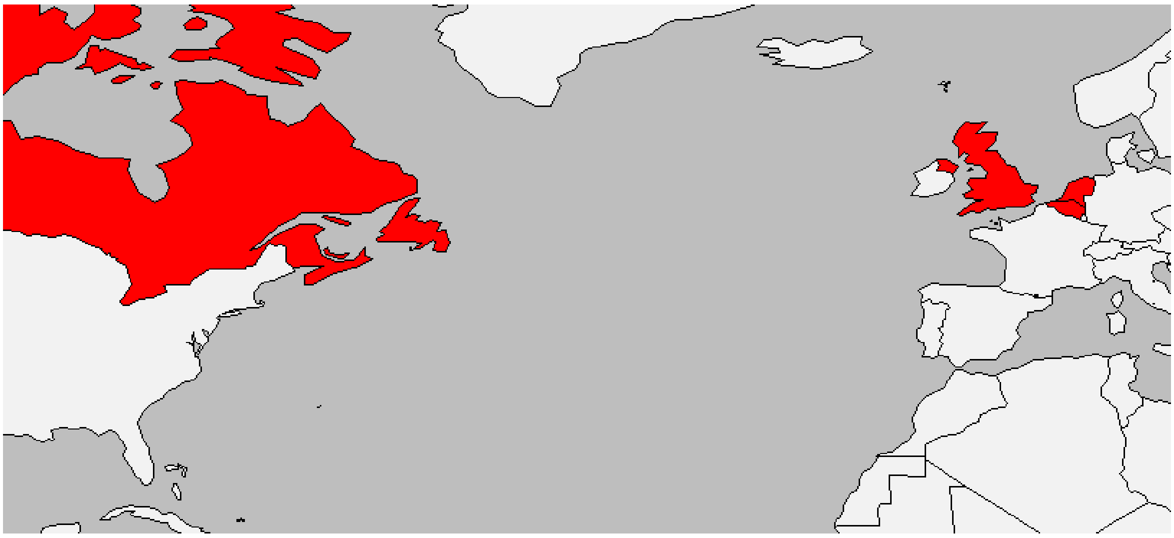

Figure 7: Illustration of a Club, Monthly Data, $H_0 : \hat{e}_t \sim I(0)$

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